

SESSION 4

Binary and Hexadecimal number systems

AOLME Curriculum Level 1

GOALS

1. Develop connections and number sense across decimal, binary, and hexadecimal systems.
2. Identify real-world applications of binary numbers.
3. Convert number values across systems.



Activities:

4.1. How do Decimal & Binary Numbers work?

4.2. Comparing Binary and Decimal Numbers

4.3. Hexadecimal Numbers

1	2
3	4

Each activity includes 1 card. One side of the card is in Spanish and on the back the same information is in English. Each card has 4 quadrants, each quadrant includes a task related to the main goal of the activity. The numbers in the square on the left describe the order to perform the tasks. The card must be at the center of the table. Students need to have access to it and take turns reading it. They can read it in the language they feel more comfortable.

4.1. HOW DO DECIMAL & BINARY NUMBERS WORK?

Activity 1 Goal:

Develop connections and number sense across decimal, binary, and hexadecimal systems.
(b) Convert number values across systems.

Resources for the Activity

- Activity Card
- Base-ten blocks and foam format to review decimal numbers
- Blocks for binary numbers
- Tables for binary numbers
- Student journal

Interactions

In this activity students are to think about the components of a computer. It is important that they recall what they remember from their prior contact with computers. Always use their experiences as a starting point and then build on ideas. While in the guidelines for this activity there is four tasks, these tasks only describe ideas and how they could be processed. Throughout the activity provide a friendly environment, supporting the participation of everyone. Notice who participates more or less and pay attention to why it might be and act on it, so participation can be more even from everyone. Support at all times the use of the language (Spanish or English) that the students want to use.

Activity Card 4.1:

4.1. How do Decimal & Binary Numbers work?

1. Decimal Numbers: We use base-ten or decimal numbers all the time. For example, we are in the Year 2019. How would represent 2019 using blocks?

Each symbol, number, or digit has a value

Thousands Hundreds Tens Ones

How come Decimal numbers grow in groups or exponents of ten?

2. Investigate: What's wrong with the picture?

The place of a number affects its value!!
In your team, take turns coming up with a decimal number and representing it using the blocks as the picture on the right shows. Describe why you did it.

Hundreds Tens Ones

3. Think Binary Numbers: What do you know about binary numbers? Why are these binary numbers?

Decimal number system uses digits (they take values from 0 to 9).

DIGIT
0 1 2 3 4 5 6 7 8 9

BIT
0 1

Binary number system uses bits or binary digits (they take the value of 0 or 1).

4. Explore: Binary numbers with only ones and zeros create any value. Explore patterns of how the blocks representing binary numbers grow. How are the patterns and exponents related?

2^5 ($2 \times 2 \times 2 \times 2 \times 2 = 32$)	2^4 ($2 \times 2 \times 2 \times 2 = 16$)	2^3 ($2 \times 2 \times 2 = 8$)	2^2 ($2 \times 2 = 4$)	2^1 ($2 \times 1 = 2$)	2^0 ($2 \times 0 = 1$)

What's an exponent?

Bit: Unit of information that a computer uses to process and retrieve data.

Complete this table using blocks. Zeros **turn off** & Ones **turn on** the values of the column wherein they are placed. **Why do you think it is so?** For example, the binary number of 101_2 is equivalent to the value of the decimal number of 5_{10} . **Why?**

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Recommended Steps for the Activity

Task 1: Have students describe what they know about decimal numbers. Ask them for examples of numbers and break them down by place value. Do not lecture, talk about it.

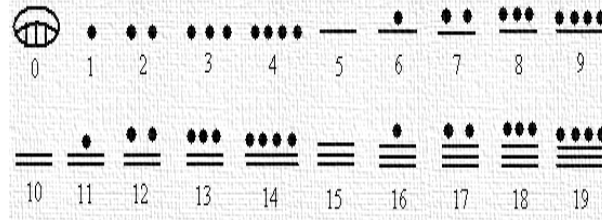
Task 2: Use foam table to represent numbers, think about place value and write numbers. Highlight how in the tens column the digits are multiplied by 10 and in the hundreds the digits are multiplied by 100. Highlight how 10^1 equal.

Task 3: This task introduces the binary numbers by describing the number of digits that there are in decimal and binary systems. Ask students to think how the digits work in decimal numbers and consider that idea to write binary numbers. For example, ask them: using only 0 and 1 how would write the value of zero, one, then how would you represent 2? With this question you can start introducing the table of binary numbers.

Task 4: Have students represent the value of each placement in the binary table using blocks. Only guide them by showing them how in the first column 2^0 equals to the value of one and that is why we have one-unit block. The next column, 2^1 the value doubles because making connections across computer systems. Use the Raspberry Pi!!

Content: Binary Numbers

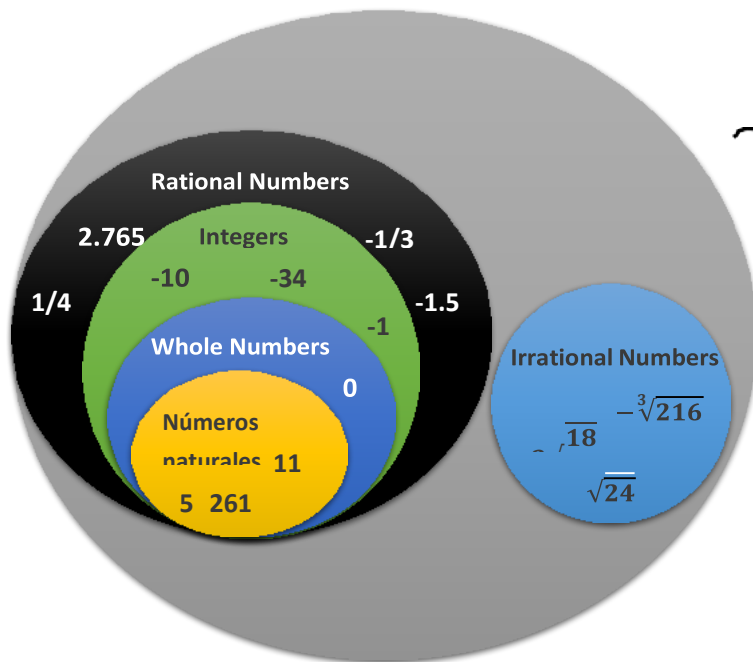
Number Systems across cultures: *Humans use different number systems to represent quantities and communicate them with each other. The numbers we use are base ten. Other numbers, like the Mayan, are base 20. In the picture above you can see the Mayan digits in base 20.*



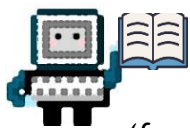
Mayan/Mesoamerican Numbers

Number Systems in Mathematics

In your math class, you have learned several number systems. In the image below, there are some examples of number systems.



Which number formats do you recognize? Share with your team

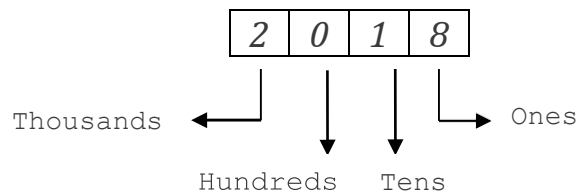


Decimal Number System

The Decimal number system is a base-10 system because it has 10 digits (from 0 to 9). The positions of the digits affect their value according to a system of 10. For example, when a 5 is in the column of ones, it is worth 5 ones or five. If the 5 is in the column of hundreds, then it is worth 5 hundreds or five hundred. Another example is the number 2,017, in which the 7 is worth seven ones, the 1 is worth ten, the 0 is worth zero hundreds, and the 2 is worth two

thousand.

Year 2018



In the table below, notice that each column relates to a specific power of the base



Although these are different systems, they use similar symbols and value system. The numbers we commonly use are called decimals because their positions are linked to a system of ten. The number 22, for example, represents 2 groups of ten and 2 ones. That means a value of twenty-two!

10 system. Look at the illustrations and write in the blank space the number that is represented below. Use the illustrations to figure out the digit for each column.

What if you added 1 more block to the column of ones? How will this number change? Write it below

thousands hundreds tens units

Write the new number using powers of 10:



_____ = ___x10³ + ___x10² + ___x10¹ + ___x10⁰

Thousand 10 ³	Hundreds 10 ²	Tens 10 ¹	Ones 10 ⁰

Complete:

If the **Decimal** Number System is "**Base 10**",

The **Binary** Number System is



How much is each of these
worth: 10^3 , 10^2 , 10^1 , 10^0 ?

BINARY NUMBER SYSTEM

The binary system is base 2! That means it uses only 2 digits.

Binary numbers are practical as they are used by digital computers.


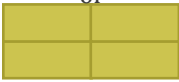






Decimal number system uses **digits** (they take values



Binary number system uses **bits** or binary digits (they take the value of 0 or 1).

Bit: Unit of information that a computer uses to process and retrieve

Binary numbers use bits or only two digits. Look at the table below. Using a sheet of paper, represent with blocks the values of the exponents given in the table below, and make sure to complete the missing representations of the exponents 2^4 and 2^5 .

2^5	2^4	2^3	2^2	2^1	2^0
		 or  or 	 or 	 or 	

Using cards of binary digits (1 and 0) place either of them in the empty spaces below of table and think and share with your team the value that you think each digit takes depending on its position.

4.2. COMPARING BINARY AND DECIMAL NUMBERS

Activity 2 Goals:

- (a) Describe how information flows in a computer system.
- (b) Practice assembling components and cables of a computer system.

Resources for the Activity

1. Activity Card
2. Raspberry Pi kit per group
3. Power strip
4. Monitor

Interactions:

Throughout the activity provide a friendly environment, supporting the participation of everyone. Notice who participates more or less and pay attention to why it might be and act on it, so participation can be more even from everyone. Support at all times the use of the language (Spanish or English) that the students want to use.

Activity Card 4.2:

4.2. Comparing Binary and Decimal Numbers

1. Practice: Using the same table, blocks and binary digits (1 & 0) represent binary numbers taking turns. Then, represent the same value using decimal blocks/numbers.

2^5 ($2 \times 2 \times 2 \times 2 \times 2 = 32$)	2^4 ($2 \times 2 \times 2 \times 2 = 16$)	2^3 ($2 \times 2 \times 2 = 8$)	2^2 ($2 \times 2 = 4$)	2^1 ($2 \times 1 = 2$)	2^0 ($2 \times 0 = 1$)

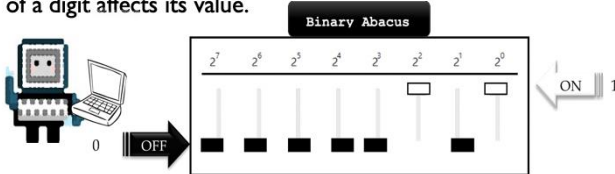
Play: Come up with a binary number and have your peers find out its value.

2. Compare: These numbers look the same, but they don't have the same value. On the left are decimal numbers and on the right binary.

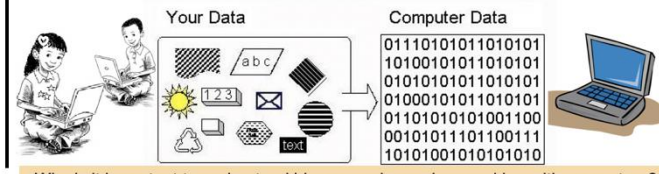
10^2	10^1	10^0	2^2	2^1	2^0
1	1	0	1	1	0

Which is worth more?

3. Represent Binary Abacus with the Binary Abacus on your laptop desktop. Take turns creating binary numbers by switching values **on** (1) or **off** (0). Write them down and justify how place of a digit affects its value.



4. Practice: Using the handout: **Creating, Writing and Comparing Numbers**, work in pairs and compare results of conversions. Check answers in Binary Abacus.



Why is it important to understand binary numbers when working with computers?

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Recommended Steps for the Activity

Task 1: Have students take turns to 'create' a new binary number by placing either the blocks or zeros and ones on the table. Elaborate on numbers created by having them or others justify why that makes sense.

Task 2: Motivate students to compare numbers that look alike make sure they see how the system (binary or decimal) give different value to the number and its position.

Task 3: Make sure all students participate in taking turns playing with the abacus. Have them predict decimal numbers.

Task 4: Have students describe their own experiences using binary numbers and think about examples provided. Practice conversion of numbers using the handout, make them support each other in solving and revising results, make sure they justify their ideas. This is a chance to check for understanding.

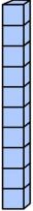
Ask for any questions they might have and encourage team members to respond.


Content: Comparing Decimal and Binary Numbers.

Let's explore values of numbers using binary and decimal systems. If needed, use blocks.

How much is **10** worth?

We know that:

In the Decimal System 10 is...	
10^1	10^0
1	0
	

In the Binary System 10 is...	
2^1	2^0
1	0
	

What are your answers, then?

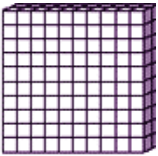

For decimals, 10 is worth: _____ For binary, 10 is worth: _____



What if the number you had was 11? What would it be worth in each system?

For decimals, 11 is worth: _____ For binary, 11 is worth: _____

One more: How much is **110** worth?

We know that:

In the Decimal System 110 is...		
10^2	10^1	10^0
1	1	0
		

In the Binary System 110 is...		
2^2	2^1	2^0
1	1	0
		

What are your answers, then?

For decimals 110 is worth: _____ For binary 110 is worth: _____

What if the number you had was 111? What would it be worth in each system?

For decimals, 111 is worth: _____ For binary, 111 is worth: _____



Discuss with your classmates.

- ⊕ When representing binary numbers, what is important to remember?

*All decimal **numbers** you have learned (e.g., 0,1,2,3,...,9) can be converted into binary digits (bit: 0s or 1s). Complete the table below adding the missing binary values that correspond to the decimal values. You may use blocks.*

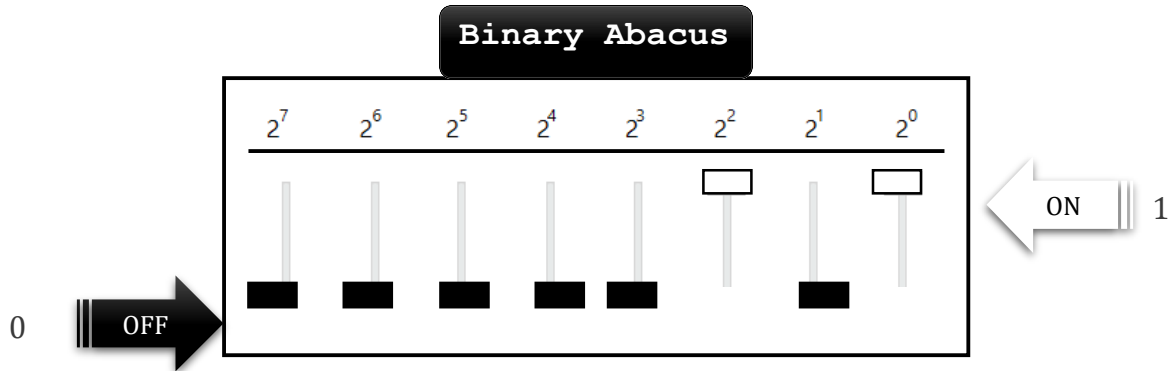



Using the table with binary numbers, blocks, and cards of binary numbers, represent the decimal and binary numbers that are missing and that are equivalent to the ones the opposing column. *Suggestion:* when representing the numbers, start with lower number then add one more block at a time to move to the higher numbers.

Decimal Numbers	Binary Numbers
0	0
1	1
2	10
3	
4	
5	101
6	
7	111
8	
9	1001
10	
11	
12	
14	1110
16	10000
20	
32	
40	
Pick a number:	
Pick a number:	

Números Decimales	Números Binarios
	10001
	10101
	1111
	100
	11

Find the **Binary Abacus** program on your laptop desktop and play with the switches **on** (1) and **off** (0).



 Discuss with your group and write down your observations.

Look at the binary numbers, what do you notice when...

- ⊕ ...all the switches are off?.....
- ⊕ ...all the switches are on?.....
- ⊕ ...some switches are on and others off?.....

Use this binary abacus to check your answers from the previous page.

We can also convert a decimal number into a binary number using mathematics. Let's look at these examples and verify if the binary numbers given to us were correct.

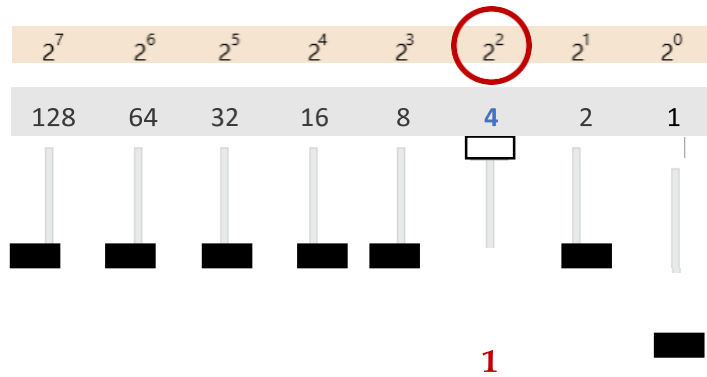
Decimal number	Binary number
5	101
46	101110

Decimal number 5:

Binary Abacus

⊕ What is the power of two closest and less than 5?

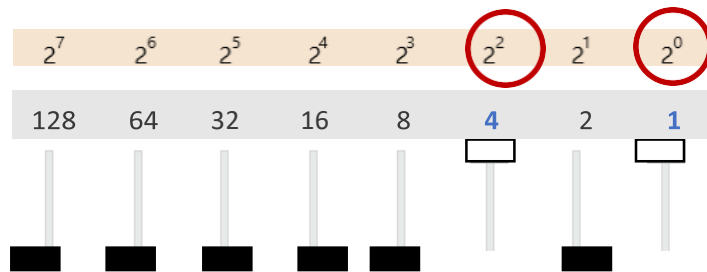
2^3 or 2^2 ?
 2^3 , no! because the result is 8,
 which is greater than 5.
 2^2 , yes! because the result is 4.
 $4 < 5$.



⊕ Then, we turn ON the third switch (2^2) by writing **1**.

⊕ We have 4 so far, but we need to get 5.

How much do we need to add?
 1



⊕ Which switch do we need to turn ON to get 1: 2^0 or 2^1 ?

Yes, $2^0 = 1$.

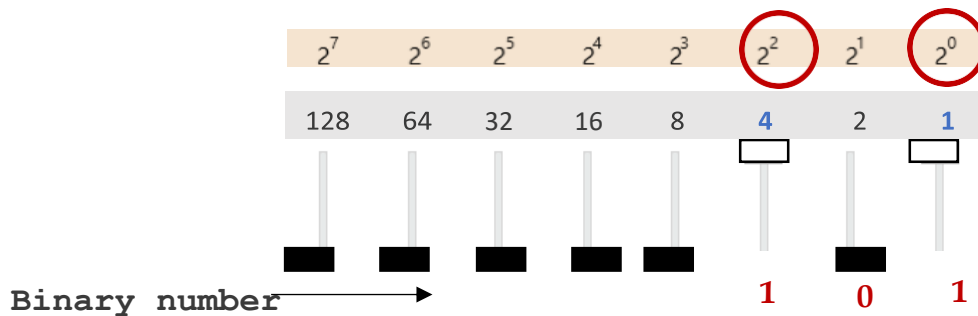
⊕ Then, we turn ON the first switch (2^0) by writing **1**.



⊕ So far the only switches ON are 2^2 and 2^0 which is equal to $4 + 1$, giving a total of 5.

⊕ All the other switches remain OFF (zero).

⊕ Once we obtain the number we want, we complete the new binary number with zeros.



Number 5 = **101** (one zero one bits)

Decimal number 46:

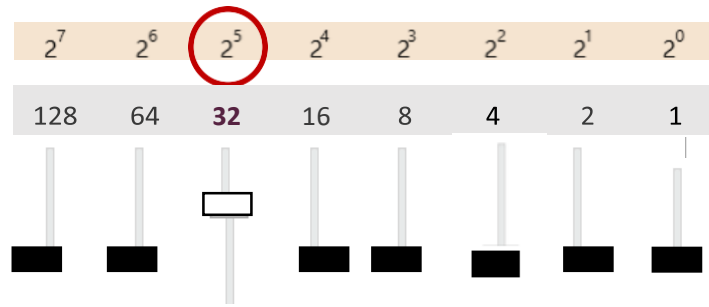
Binary Abacus

⊕ What is the power of two closest and less than 46?

2^6 or 2^5 ?

2^6 , no! because the result is 64, which is greater than 46.

2^5 , yes! because the result is 32.
 $32 < 46$.



⊕ Then, we turn ON the sixth switch (2^5) by writing **1**.

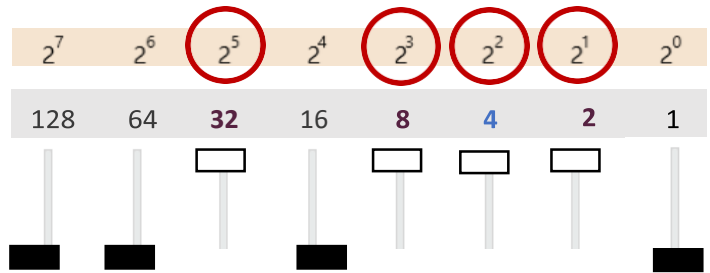
⊕ We have 32 so far, but we need to get 46. How much do we need to add? 14

⊕ Which switches do we need to turn ON to get 14? 2^0 , 2^1 , 2^2 , 2^3 , 2^4 ?

Yes, 2^1 , 2^2 and 2^3

⊕ Why?

Because $2^1 + 2^2 + 2^3 = 2 + 4 + 8 = 14$.



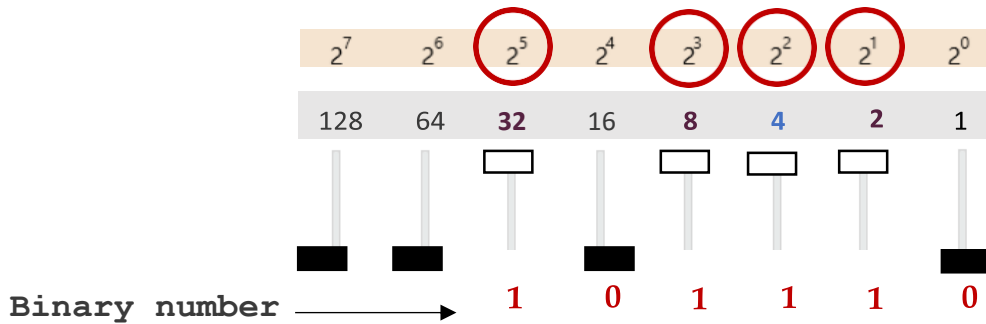
⊕ Then, we turn ON the second (2^1), third (2^2), and fourth (2^3) switch by writing 1.

1 1 1 1

⊕ So far the only switches ON are 2^1 , 2^2 , 2^3 and 2^5 which is equal to $2 + 4 + 8 + 32$, giving a total of 46.

⊕ All the other switches remain OFF (zero)

⊕ Once we obtain the number we want, we complete the new binary number with zeros.



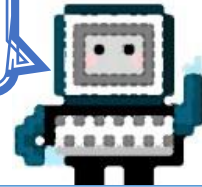
Number 46 = 101110 (one zero one one one zero bits)





Binary numbers in life scenarios

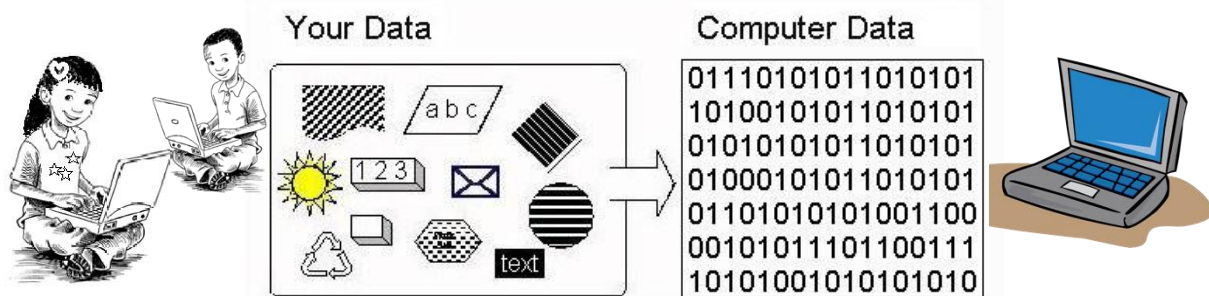
But where do we use binary



In addition to computers, barcodes, which are often found on self-adhesive or machine-printed envelopes, can be decoded into 0's and 1's. Those black and white lines contain a lot of information (e.g., address, ZIP codes, product, etc.)

Computers understand everything in 0s and 1s. Numbers, texts, graphics, images, videos, all you type and see in a computer is converted into **BINARY CODES**.

BINARY CODES → system of numbers based on two possible bits 0 and 1 (e.g., 0011100100010100010)



- ⊕ Electronics can only understand two distinct types of data: **on** and **off**.
- ⊕ **Binary code is the language of the computer** and it is represented by a collection of **on/off** or 1s and 0s bit (Computer Scientists and Engineers represent each bit 0 or 1 as the state of a transistor.)

1bit = transistor ON 0 bit= transistor OFF

4.3. HEXADECIMAL NUMBERS

Activity 3 Goal:


Utilize basic Linux commands to navigate filesystems in a Raspberry Pi.

Resources for the Activity

1. Activity Card
2. Raspberry Pi kit
3. Student journal

Activity Card 4.3:

4.3. Hexadecimal Numbers



What base are these number notations in?
 111_2 $A0_{16}$ 1010 $1A_{16}$
 Create your own number system notation!

1. "Hexadecimal" means Hexa-(six) plus decimal (ten), or sixteen, or "based 16". People also call them Hex. Hex numbers have 16 hexadecimal digits (or nibbles). To represent them we use the symbols below:

How would you write 16 in Hex?

HEXADECIMAL DIGIT																
	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
Decimal value:	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

These are some Hex numbers: 51_{16} $A3_{16}$ $F8_{16}$ BD_{16}

2. Hex grow in groups of 16. What's the value of $2A_{16}$ and $A2_{16}$?

16^1	16^0
10 blocks	2 blocks

Take turns, using the hex blocks, digits, & table (red sheet, represent hex numbers. Remember, place matters!

Use decimals numbers to write on board the value of each number:
 10 (decimal): __, 10_2 (binary): __, 10_{16} (hexadecimal): __
 11 (decimal): __, 11_2 (binary): __, 11_{16} (hexadecimal): __

3. Conversions: Binary → Hexadecimals
 Hex help write binary numbers easier by grouping binary numbers in bits or sets of 4 binary digits. Each bit can be represented with a hex. For example, 101110_2 :

Using the table on the back of the Binary table, take turns converting binary into Hex numbers. Follow similar procedures to these ones:

Binary	<u>0010</u> <u>1110</u>	
Decimal Value	2	14
Hexadecimal	2	E

Why do you think these procedures work?

4. Challenge: Now that you have learned new numbers systems, complete the table called: "Table of Three Numbers Systems from 0-15". This table might help remember in the future how these numbers work.

Write about what you learned in Session 4. For example, write sentences that start with:
 -Digits of binary numbers are ...
 -Exponents are numbers that ... They mean ...
 -Hex and binary numbers are different in...

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Recommended Steps for the Activity

Task 1: Have students think about how base 16 works in groups of 16, so we can use the hex digits to make these numbers.

Task 2: Compare how the column on the left is groups of 16 and on the right are units. Use blocks to help represent this idea.

Task 3: Motivate students to take turns converting binary into hex. Make sure they understand how the binary numbers when separated into groups of 4 digits, the positions of the numbers change also, so for example the ones on the left, the first number is not 2 to the fourth power but 2 to de zero power.

Task 4: Have students complete the table comparing decimals, binary and hex, ask them to find patterns and count in each system. Debrief what they learned at the end of the session and write in their journal at least 3 thoughts.

Content: